Models of Pilot Wave Quantum Field Theory by Matthew Dickau
A response

1 Introduction

I have recently been discussing pilot wave interpretations of quantum field theory with a commentator Matthew Dickau. My initial thoughts on the subject were fairly sparse. Like most physicists, I rejected the pilot wave model because of its explicit non-locality, violation of Lorentz invariance, and failure to incorporate particle creation and annihilation. Matthew has challenged me on this. He favours the pilot wave interpretation because of its explicit realism. For my own work, this is a serious issue. My own philosophy and work relies on quantum indeterminacy, and Bohm's interpretation is one of two I know of (the other being the Everett interpretation) which accepts a fully deterministic physics.

Matthew has responded by firstly challenging me, via Bell's theorem, that there must be some non-locality in the physical interpretation, and secondly that there are a number of researchers attempting to reconcile quantum field theory with a pilot wave interpretation. None of these researchers have yet got to a point where they can definitively reproduce the results of standard QFT, but they are making some progress.

In addition, Matthew has drafted a paper[1], which attempts to create a pilot theory interpretation of various QFT theories. This article (which I will expand as I proceed) is critique of that work, intended more in the spirit of offering a few thoughts which will hopefully direct him towards refining his work.

There are three sections in the article. The first outlines classical field theory, and is not controversial. There are then two models of a pilot wave quantum field theory: a 1+1 dimensional scalar field theory, and a 2+1 dimensional gauge theory.

My intention in responding to each section is to first of all summarise the content of that section, and then to note down any thoughts I had as I read through the paper.

2 Preliminaries - Classical Field Theory and Quantum Theories

2.1 Summary

In this section, Matthew gives a brief outline of what he means by a classical field theory and a quantum theory. He introduces the Lagrangian and Hamiltonian, and their use in classical mechanics in deriving the equations of motion. He distinguishes a field from a particle in that a field associates a scalar, vector or other quantity with each point in space time. In field theory, the Lagrangian can be written as an integral of a Lagrangian density over all space. He also distinguishes between free and interacting theories. He defines the mass of a field via its (relativistic) dispersion relation.

He then discusses quantum physics, which becomes important at scales close to $\hbar$, where the particle/field distinction of classical physics seems to break down. He states that physicists are not usually consistent in their mathematical treatment of quantum physics, sometimes treating the quantum state as an objectively real entity, and other times as reflecting the probability for the system to be in a given state. While the predictions of the theory are very accurate, it seems confused from the perspective of scientific realism. In contrast, the pilot wave interpretation of quantum mechanics is clear. There is an actual physical state of the system in addition to the quantum state. The physical state can be described by some coordinates, q_i. The quantum state $\psi(q_i, t)$ is a function of these coordinates. Both of these are well defined at all times. The quantum

[1] https://structureoftruth.files.wordpress.com/2020/05/models-of-pilot-wave-qft.pdf

state (which we cannot observe directly) evolves according to Schrödinger's equation,

$$i\hbar\frac{\partial\psi(q_i,t)}{\partial t} = \hat{H}\psi(q_i,t). \tag{1}$$

In turn, the wavefunction directs the evolution of the actual configuration,Q, (which we can measure) according to a guidance equation, which in the cases we are interested in will be

$$\frac{\partial Q_i}{\partial t} = \mathrm{Re}\frac{i}{\hbar}\frac{\psi^\dagger(Q_i,t)[\hat{H},Q_i]\psi(Q_i,t)}{\psi^\dagger(Q_i,t)\psi(Q_i,t)} \tag{2}$$

He takes the view that a QFT is a quantum theory of fields rather than a theory of particle creation and annihilation (which is my own view).

2.2 Comments

1. In his discussion of the classical dispersion relation for fields, Matthew makes use of the relationship $E = \hbar\omega$. This is actually part of quantum physics rather than classical physics.
2. In his discussion of the pilot wave guidance equations, the functional dependence on q and t is written in a strange and confusing notation; it should be as I presented it above.
3. In his discussion of interacting field theories, he states that the superposition principle doesn't apply. He needs to make clearer what he means by the superposition principle here. Usually, it is taken to mean how amplitudes rather than intensities of two different waves add up in (for example in electromagnetic waves). So, for example, two large amplitudes can cancel each other out leading to zero intensity. He states that in interacting field theories, the superposition principle is removed. This isn't, I think, correct: certainly as I have defined the principle. It will certainly be modified by the interactions, but you will still be adding up the amplitudes, and squaring the sum to get an intensity.

3 The first model: 1+1 dimensional scalar fields

3.1 Summary

In this section, Matthew presents a one (plus one) dimensional scalar field theory as a toy model. His model exists on a spatial torus of circumference l with a continuous time. A real scalar field can therefore be decomposed as

$$\phi(t,x) = \sum_{n=-\infty}^{\infty} C_n(t)e^{ik_nx} \qquad C_{-n}(t) = C_n^\dagger(t) \qquad k_n = 2\pi n/l \tag{3}$$

The degrees of freedom can be made finite rather than infinite by restricting the number of degrees of freedom by taking the sum from $-N$ to N. Although Matthew doesn't mention this, this has the effect of turning the theory from a continuum theory to a lattice theory, with the scalar field only defined on lattice points separated by a fixed lattice spacing. The lattice spacing l/N and momentum cut-off $1/l$ break Lorentz invariance, but Matthew believes that this will be restored in the limits $N \to \infty$ and then $l \to \infty$.

The Lagrangian Matthew is going to consider consists of a complex scalar field, ψ, a real scalar field, ϕ, and an interaction term, with a Lagrangian density

$$\mathcal{L} = \partial^\mu\psi^\dagger\partial_\mu\psi - m^2\psi^\dagger\psi + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\mu^2\phi^2 - g\psi^\dagger\phi\psi. \tag{4}$$

The mode expansion is defined as

$$\phi(t,x) = \sum_{n=-N}^{N} \frac{1}{\sqrt{l}} Q_n(t) e^{ik_n x} \qquad Q_n(t) = Q^\dagger_{-n}(t) \tag{5}$$

$$\psi(t,x) = \sum_{n=-N}^{N} \frac{1}{\sqrt{l}} q_n(t) e^{ik_n x} \tag{6}$$

He further decomposes these parameters into real degrees of freedom,

$$q_n = \frac{1}{\sqrt{2}}(u_n + iv_n) \tag{7}$$

$$Q_n = \frac{1}{\sqrt{2}}(U_n + iV_n) \tag{8}$$

From these, he constructs the Lagrangian in terms of these variables (in total, there are $6N + 3$ variables, given the restrictions on U and V from the condition that the ϕ field is real). The canonical momenta are defined according to

$$\pi_{X_n} = \frac{\partial X_n}{\partial t}, \tag{9}$$

where X_n represents either u_n, v_n for $-N \leq n \leq N$, U_n for $0 \leq n \leq N$ or V_n for $1 \leq n \leq N$.

From this Matthew derives the Hamiltonian of the classical field theory, which is essentially just the Hamiltonian of $6N + 3$ coupled Harmonic oscillators.

To quantise the theory, Matthew takes this set of functions as his configuration space, and introduces a wavefunction $\Psi(U_n, V_n, u_n, v_n, t)$. He converts the fields into operators acting on the wavefunction according to

$$\hat{X}_i \Psi = X_i \Psi \qquad \hat{\pi}_{X_i} \Psi = -i \frac{\partial}{\partial X_i} \Psi. \tag{10}$$

Then Ψ evolves via the Schrödinger equation,

$$i \frac{\partial \Psi}{\partial t} = \hat{H} \Psi \tag{11}$$

and the coordinates evolve as,

$$\frac{\partial X_i}{\partial t} = \frac{1}{2i|\Psi|^2} \left(\psi^\dagger \frac{\partial \Psi}{\partial X_i} - \Psi \frac{\partial \Psi^\dagger}{\partial X_i} \right) \tag{12}$$

Matthew goes on to consider the free theory, and in particular to construct the energy eigenstates. Since this is just a set of Harmonic oscillators, the energy eigenvalues are $(n + \frac{1}{2})\hbar\omega_i$ where $\omega_i = \sqrt{k_i^2 + m^2}$. These come in discrete lumps, which Matthew interprets as particles. He then proposes an algorithm where the interacting theory might be solved numerically.

3.2 Comments

1. The Lagrangian is written using a continuum spatial derivative. This is incorrect in a lattice theory. You need to define a discrete derivative operator (and ensure that it lacks any doublers - spurious modes of zero energy).

2. I think the Noether current at the bottom of page 6 is incorrect. I got $j_\mu = i\partial_\mu(\psi^\dagger)\psi - \psi^\dagger\partial_\mu(\psi)$

3. The probability density for a Klein Gordon theory is $i(\psi^\dagger \partial t \psi - \partial t(\psi^\dagger)\psi)$, not the expression at the top of page ten.

4. Note that the coordinates Q and q are effectively Fourier transforms of the real scalar fields. These are thus (in the quantum theory) momentum fields. π would thus correspond to location fields, somewhat confusingly.

5. I am concerned about the quantisation of the "momentum" field, $\hat{\pi}_{X_i}\Psi = -i\frac{\partial}{\partial X_i}\Psi$. Firstly it should be noted that the derivative is a functional derivative rather than a standard derivative (u and v are functions rather than scalar numbers). This doesn't matter too much here, but will introduce complications if you try to expand this to Fermions. Equally, the replacement of the "momentum" operator with a functional derivative is not standard. Don't forget this isn't a momentum - your momentum is k_n, so the usual analogue with the standard quantum mechanical substitution $\hat{P} \to -i\frac{\partial}{\partial x}$. The usual thing to do in canonical quantisation is to replace $\hat{\pi}$ and $\hat{X}$ as operators, and impose commutation relations on them. You maintain the correct commutation relations, so this might be OK, but I am still a but nervous. In standard quantum field theory, the degrees of freedom would be the ψ and $\Pi = \partial_t\psi$ fields. You would impose commutation relations on these, and use that to construct creation and annihilation operators. You need (in effect) to be doing the same thing here. I am not certain that it is wrong (in this case), given that your coordinates are just the Fourier transforms of the original fields, but I am a bit nervous here. You might not be reproducing the same theory as the straight-forward quantisation of the original Lagrangian.

6. In your derivation of the Energy levels of your Hamiltonian, I think it would be better if you explicitly constructed the raising and lowering operators for your Hamiltonian. Firstly, it would be a more explicit way of showing the energy levels. But more importantly, after re-writing your Hamiltonian in those terms, it will allow you to reproduce the standard QFT formulation for this theory.

7. I am not sure of the role that your coordinates U, V, u, and v play in this theory. They don't represent observable particles, since the raising operator (which represents particle creation) will be a linear combination of U and π_U. In what way do these coordinates help me make predictions from this theory?

8. I would recommend that rather than your numerical approach to solve this system, you first of all try using perturbation theory. This would allow you to directly compare against the results of standard QFT. So start by considering the free field theory, and then systematically add in corrections of higher orders of the coupling constant g.

9. To convince any physicist, you are going to have to show that the predictions of this theory match those of the standard formulation of QFT. If you can calculate a cross section for a simple observable to the first few orders in perturbation theory in both standard QFT and this theory, then it would be more convincing. That's what I as a physicist am most interested in.

10. I think you have been too naive when you say that Lorentz invariance will be restored in the $N \to \infty$, $l \to \infty$ limit. It is not as simple as that. When taking a limit, your goal is to show that some parameter gradually approaches a limiting value as you move towards the limit. So in this case, you would need some numerical parameterization of the degree of breaking of Lorentz symmetry, and show that goes to zero as you take your limits. I don't think that this will be possible. For every finite N and l, you break the symmetry in exactly the same way. So I don't think that your claim that the symmetry is restored is valid. It needs to be proven. You can have lattice field theories, and you can have a continuum limit of them, but it is a far more complex process than this, involving the renormalisation group.

11. How do you define your length scale l? In what units is it measured?

12. I am concerned that you haven't given any thought to renormalisation. While I haven't done the calculation, I would be very surprised if this model (using the standard QFT calculation)

didn't require regularisation and then renormalisation to obtain the correct results and avoid infinities. How does that fit into your model?

13. I am also not convinced that this is a quantum field theory. It more resembles a set of coupled quantum mechanical harmonic oscillators. You might say that it becomes a field in the appropriate limits, but taking limits is always a dangerous occupation; you need a firm proof. In particular, everything in the theory evolves smoothly according to differential equations. I would not expect any discontinuities in the solutions. The problem with this smooth evolution with no discontinuous jumps is that you are not going to sample any different sectors of the theory; whether those sectors are defined by different particle number, or different homotopy classes when you come to non-Abelian gauge theory. How would particle creation manifest itself in this theory, in particular with regards to the evolution of U, V and so on?

4 The second model: 2+1 dimensional gauge theory

4.1 Summary

In this section,

4.2 Comments